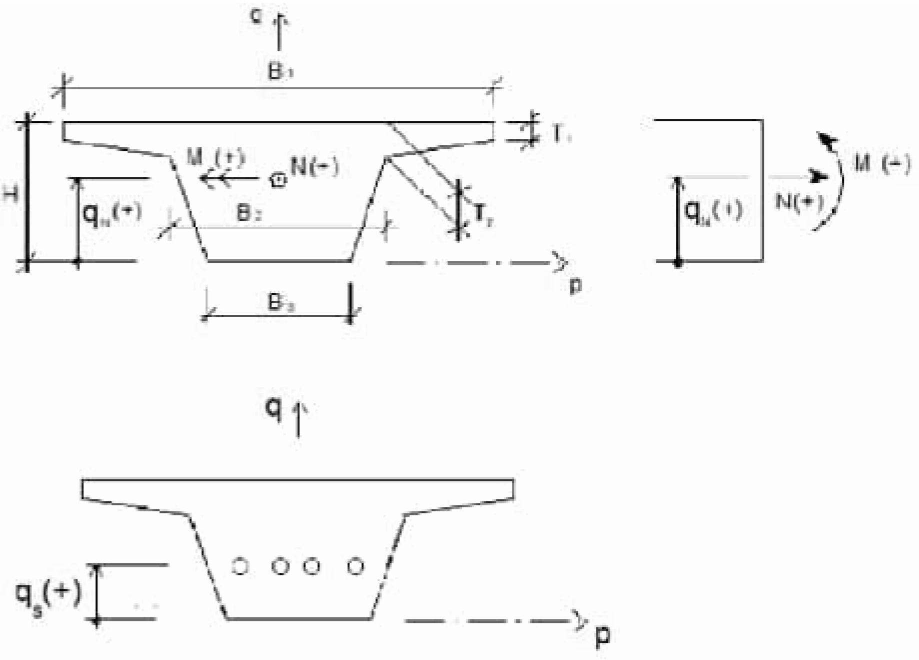
Object: Girder L1

PRINCIPLE SKETCH



THEORY

Stresses in concrete are determined for stage I (uncracked concrete) at the top of the beam ($\sigma_{c.ök}$), bottom of the beam ($\sigma_{c.uk}$) and the prestressing steel (σ_{cp}). A check is carried out to see whether cracking according to EC2-1 section 7.3.2 (4) of the National Annex occurs. This is considered to occur when tensile stresses in the transverse section (σ_{ct}) exceed f_{ctk}/ζ.

If that is the case, fatigue is studied in Stage II (cracked concrete).

When checking concrete compression stresses, $\sigma_{c.uk} = \sigma_{c.ök} = 0$ MPa shall be applied if the bending moment(M) produces $\sigma_c > 0$ MPa (i.e. tension).

When checking stresses in the prestressing steel, $\sigma_s = \alpha \cdot \sigma_{cp}$ shall be applied if the bending moment (M) produces $\sigma_{cp} < 0$ MPa (i.e. compression).

INPUT

Number of sections

$N := 2 \cdot pcs$

Number of load cycles

$N := 0.5 \cdot 10^6 \cdot cycles$

Creep

$\phi_{ef} := 2.0$

Concrete

$f_{ck} := 35 \cdot MPa$

$f_{ctm} := 2.1 \cdot MPa$

$E_{cm} := 32 \cdot GPa$

$\gamma_c := 1.50$: ultimate state ULS according to EC2-1-1 table 2.1N

$k_{1.c} := 1.00$: factor that considers time when loading starts according to SS-EN 1992-2 section 6.8.7. Value 1.00 is assumed according to TRVFS 2011:12.

$\beta_{cc} := 1$

$\zeta := 1.5$

$\alpha_{cc} := 1.00$: correction factor concrete resistance

Prestressed tendons

$\gamma_{s.fat} := 1.15$: according to EC2-1-1 table 2.1N

$E_{sk} := 195 \cdot GPa$

$f_{pk} := 1600 \cdot MPa$

$A_p := 1110 \cdot mm^2$

Section	n_p	q_s
1	2	100
2	2	100
-	pcs	mm

Geometry

$N := 2 \cdot pcs$

Section	B_1	B_2	B_3	T_1	T_2	H
1	1000	998	996	200	250	1000
2	1000	998	996	200	250	1000
-	mm	mm	mm	mm	mm	mm

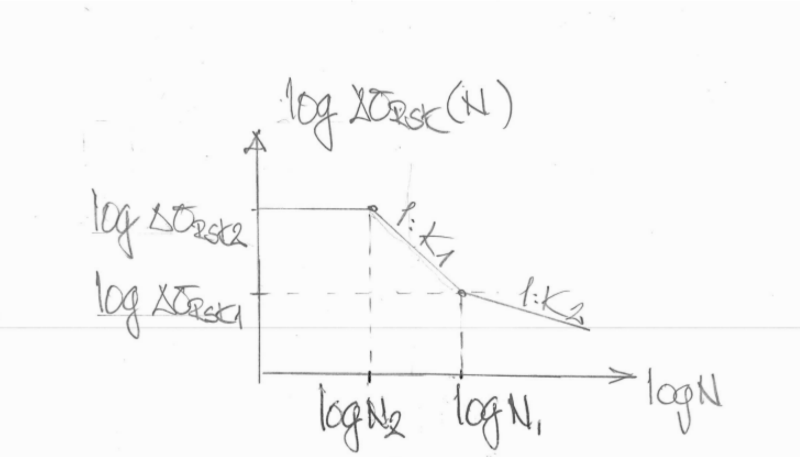
Parameter associated to Wöhler curves for pretension
(SS-EN 1992-1-1- table 6.4N)

$N_1 = N^*$

$\Delta\sigma_{Rsk.1} = \Delta\sigma_{Rsk}(N_1)$

$\Delta\sigma_{Rsk.2} = \Delta\sigma_{Rsk}(N_2)$

$\Delta\sigma_{Rsk.2} = f_{yd}$



$k_1 := 5$

$k_2 := 7$

$N_1 := 10^6 \cdot cycles$

$\Delta\sigma_{Rsk.1} := 120 \cdot MPa$

Cross section properties (stadium I):

Section	A_c	I_c	q_{TP}	
1	1,000	0,0833	500	←"field"
2	1,000	0,0833	500	←"support"
-	m^2	m^4	mm	

Forces FAT-0 (load effects fatigue excl. pretension)

M > 0 corresponds to tension at "bottom" of beam / M < corresponds to tension at "top" of beam

N < 0 corresponds to compressive normal force / N > 0 correspond to tensional normal force

Section	$M_{max.0}$	$N_{tillh.10}$	$M_{min.0}$	$N_{tillh.20}$	q_N	
1	680	0	-250	0	500	←"field"
2	680	0	-250	0	500	←"support"
-	kNm	kN	kNm	kN	mm	

Forces FÖRSP (load effects pretension incl. parasite forces)

Section	$M_{FÖRSP}$	$N_{FÖRSP}$	q_N	
1	0	-2391	500	←"field"
2	0	-2391	500	←"support"
-	kNm	kN	mm	

CALCULATION

Total tendon area

$A'_p := A_p \cdot n_p$

Forces FAT (load effect fatigue incl. pretension)

$M_{max} := M_{max.0} + M_{FÖRSP}$

$N_{tillh.1} := N_{tillh.10} + N_{FÖRSP}$

$M_{min} := M_{min.0} + M_{FÖRSP}$

$N_{tillh.2} := N_{tillh.20} + N_{FÖRSP}$

Section	M_{max}	$N_{tillh.1}$	M_{min}	$N_{tillh.2}$	
1	680	-2391	-250	-2391	←"field"
2	680	-2391	-250	-2391	←"support"
-	kNm	kN	kNm	kN	

Stress pretension

$\sigma_p := -\dfrac{N_{FÖRSP}}{A'_p}$

Fatigue resistance concrete

$f_{cd} := \alpha_{cc} \cdot \dfrac{f_{ck}}{\gamma_c} = 23.3\ MPa$: see EC2-1-1 equation 3.15

$f_{cd.fat} := k_{1.c} \cdot \beta_{cc} \cdot f_{cd} \cdot \left(1 - \dfrac{f_{ck}}{250 \cdot MPa}\right) = 20.1\ MPa$: see EC2-1-1 equation 6.7.6

E-modulus quote

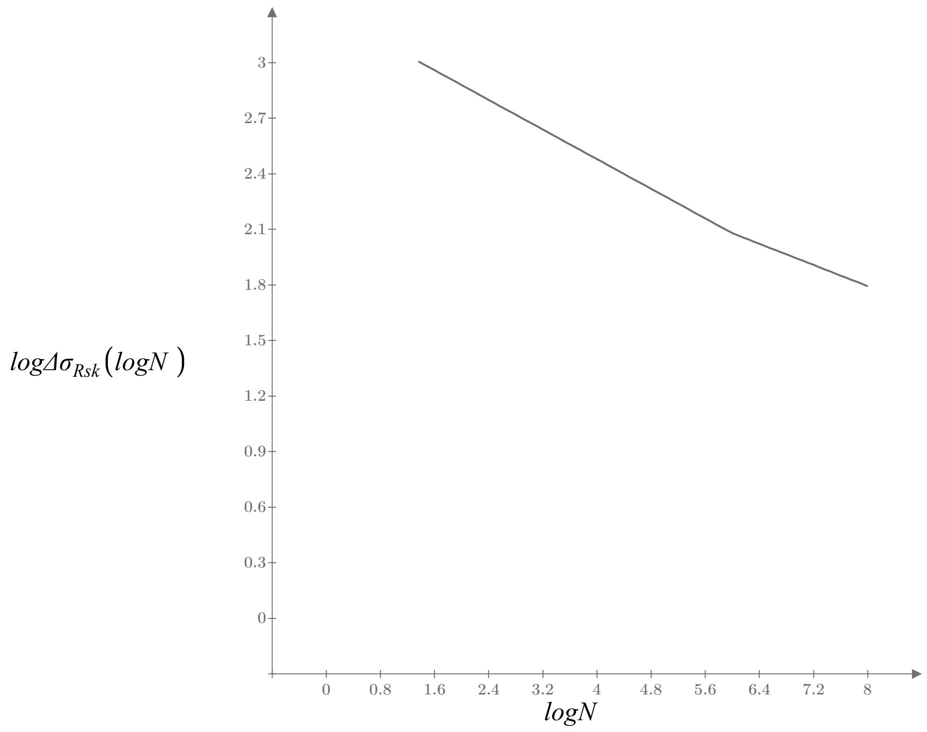
When determining neutral axis α is used, while α_e is use when considering EC2-1 equation 7.9.

$\alpha := \dfrac{E_{sk}}{E_{cm}} \cdot \left(1 + \phi_{ef}\right) = 18.3$: including creep

$\alpha_e := \dfrac{E_{sk}}{E_{cm}} = 6.1$: excluding creep

Permissible stress tendon

$$
logΔσ_{Rsk} := \left\|
\begin{array}{l}
logΔσ_{Rsk.1} \leftarrow \log\left(Δσ_{Rsk.1}\right) \\
logN_1 \leftarrow \log\left(N_1\right) \\
logΔσ_{Rsk.2} \leftarrow \log\left(f_{pk}\right) \\
logN_2 \leftarrow \log\left(N_1\right) - \left(logΔσ_{Rsk.2} - logΔσ_{Rsk.1}\right) \cdot k_1 \\
\textbf{if } logN \le logN_2 \\
\quad \left\| logΔσ_{Rsk} \leftarrow logΔσ_{Rsk.2} \right. \\
\textbf{if } logN_2 < logN \le logN_1 \\
\quad \left\| logΔσ_{Rsk} \leftarrow logΔσ_{Rsk.2} - \dfrac{logΔσ_{Rsk.2} - logΔσ_{Rsk.1}}{logN_2 - logN_1} \cdot \left(logN_2 - logN\right) \right. \\
\textbf{if } logN \ge logN_1 \\
\quad \left\| logΔσ_{Rsk} \leftarrow logΔσ_{Rsk.1} - \dfrac{1}{k_2} \cdot \left(logN - logN_1\right) \right. \\
logΔσ_{Rsk}
\end{array}
\right.
$$

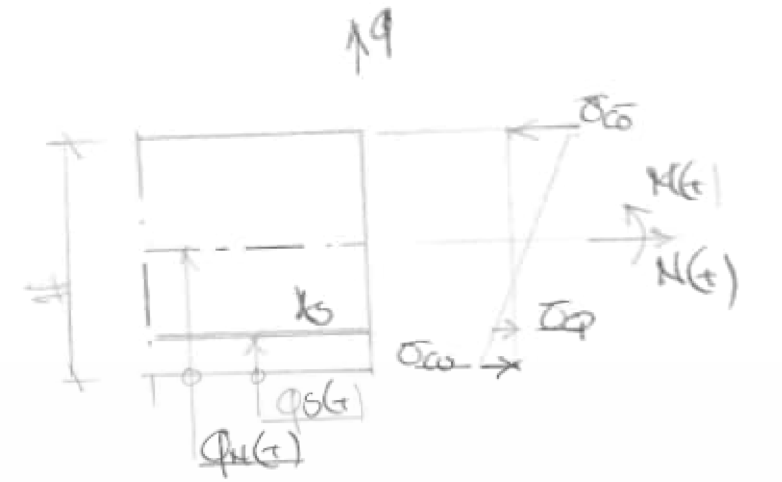


$$exp := logΔσ_{Rsk}\left(\log\left(N\right)\right) = 3.204$$

$$Δσ_{Rsk} := 10^{exp} \cdot MPa = 1600\ MPa$$

$$Δσ_{Rsd} := \frac{Δσ_{Rsk}}{γ_{s.fat}} = 1391\ MPa$$

Defintions (stadium I)



Stress in uncracked concrete (stadium I)
When σ_c > 0 MPa tensional stress occurs If σ_c < 0 MPa then compressive stress occurs.

$$\sigma_{ct.till} := \frac{f_{ctm}}{\zeta} = 1.4\ MPa$$

Maximum moment:

$$\sigma_{cö.1} = \frac{N_{tillh.1}}{A_c} - \frac{M_{max}}{I_c} \cdot (H - q_{TP})$$

$$\sigma_{cu.1} = \frac{N_{tillh.1}}{A_c} + \frac{M_{max}}{I_c} \cdot q_{TP}$$

$$\sigma_{cp.1} = \frac{N_{tillh.1}}{A_c} + \frac{M_{max}}{I_c} \cdot (q_{TP} - q_s)$$

Section	σ cö.1	σ cu.1	σ cp.1		σ cttill	Stadium verification	
1	-6,5	1,7	0,9	>	1,4	Stadium II	←"field"
2	-6,5	1,7	0,9	>	1,4	Stadium II	←"support'
-	MPa	MPa	MPa		MPa	-	

Minimal moment:

$$\sigma_{cö.2} = \frac{N_{tillh.2}}{A_c} - \frac{M_{min}}{I_c} \cdot (H - q_{TP})$$

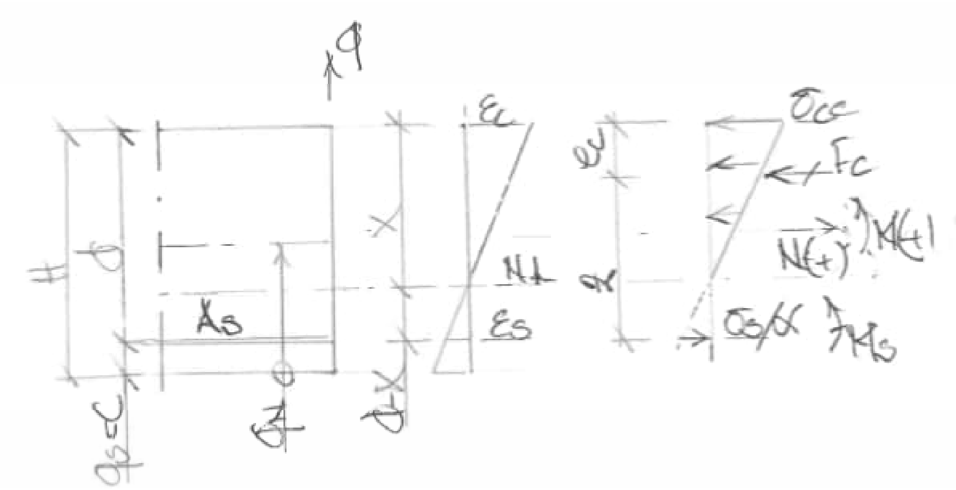
$$\sigma_{cu.2} = \frac{N_{tillh.2}}{A_c} + \frac{M_{min}}{I_c} \cdot q_{TP}$$

$$\sigma_{cp.2} = \frac{N_{tillh.2}}{A_c} + \frac{M_{min}}{I_c} \cdot (q_{TP} - q_s)$$

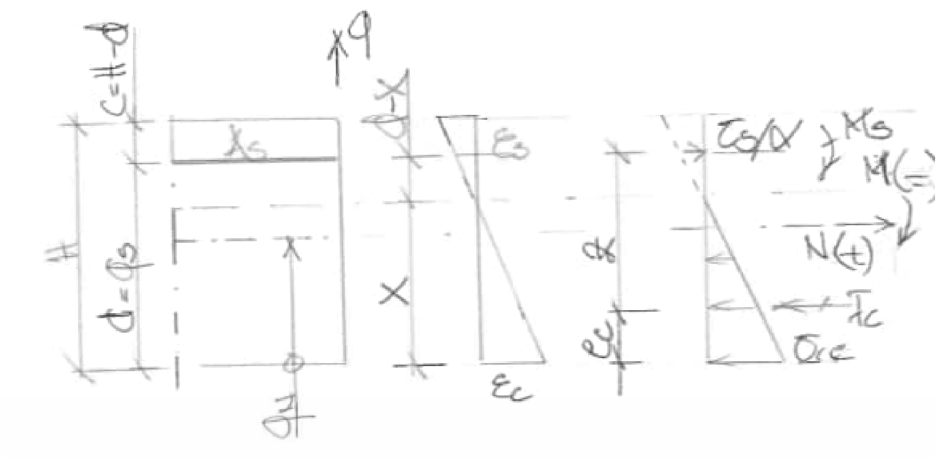
Section	$\sigma_{cö.2}$	$\sigma_{cu.2}$	$\sigma_{cp.2}$		$\sigma_{ct.till}$	Stadium verification	
1	-0,9	-3,9	-3,6	<	1,4	Stadium I	← "field"
2	-0,9	-3,9	-3,6	<	1,4	Stadium I	← "support'
-	MPa	MPa	MPa		MPa	-	

Defintion (stadium II)

Moment resulting in tension at "bottom" of beam:



Moment resulting in tension at "top" of beam:



Function - moment when normal force is moved to to center of cables (stadium II)

$$M_{s.max} = \begin{cases} M_{max} - N_{tillh.1} \cdot (q_N - q_s) & \text{if } M_{max} > 0 \cdot kNm \\ -M_{max} + N_{tillh.1} \cdot (q_s - q_N) & \text{if } M_{max} \le 0 \cdot kNm \end{cases}$$

$$M_{s.min} = \begin{cases} M_{min} - N_{tillh.2} \cdot (q_N - q_s) & \text{if } M_{min} > 0 \cdot kNm \\ -M_{min} + N_{tillh.2} \cdot (q_s - q_N) & \text{if } M_{min} \le 0 \cdot kNm \end{cases}$$

Section	$M_{s.max}$	$M_{s.max}$	
1	1636	1206	← "field"
2	1636	1206	← "support"
-	kNm	kNm	

Function - p:coordinate as function of q:coordinate

$$p = \begin{cases} 0.5 \cdot \left[B_3 + (B_2 - B_3) \cdot \dfrac{q}{H - T_2}\right] & \text{if } q \le (H - T_2) \\ 0.5 \cdot \left[B_2 + (B_1 - B_2) \cdot \dfrac{q - H + T_2}{T_2 - T_1}\right] & \text{if } (H - T_2) < q \le (H - T_1) \\ 0.5 \cdot B_1 & \text{if } (H - T_1) < q \le H \end{cases}$$

Function - effective area "compressive zon" (stadium II)

$$A_c = \begin{cases} 2 \displaystyle\int_{H-x}^{H} \frac{q - (H - x)}{x} \cdot p \, dq & \text{if } M_d \ge 0kNm \\ 2 \displaystyle\int_{0m}^{x} \frac{x - q}{x} \cdot p \, dq & \text{if } M_d < 0kNm \end{cases}$$

Function - distance to centroid effective area "compressive zon" (stadium II)

$$e_c = \begin{cases} \text{if } M_d \geq 0kNm \\ \quad S_p \leftarrow 2\int_{H-x}^{H} \frac{q-(H-x)}{x} \cdot p \cdot q \, dq \\ \quad e_c \leftarrow H - \frac{S_p}{A_c} \\ \text{if } M_d < 0kNm \\ \quad S_p \leftarrow 2\int_{0m}^{x} \frac{x-q}{x} \cdot p \, q \, dq \\ \quad e_c \leftarrow \frac{S_p}{A_c} \end{cases}$$

Function - effective height

$$d = \begin{cases} H - q_s & \text{if } M_d > 0 \cdot kNm \\ q_s & \text{if } M_d \leq 0 \cdot kNm \end{cases}$$

Function - calculation location of neutral axis (stadium II)

Maximal moment:

$$X_{max} = \left| \begin{array}{l} x_0 \leftarrow 0.5H \\ z \leftarrow d - e_c \\ F_c \leftarrow \dfrac{M_{s.max}}{z} \\ \sigma_{cc} \leftarrow \dfrac{F_c}{A_c} \\ F_s \leftarrow F_c + N_{tillh.1} \\ \sigma_s \leftarrow \dfrac{F_s}{A'_p} \\ x_1 \leftarrow \dfrac{\alpha \sigma_{cc}}{\sigma_s + \alpha \sigma_{cc}} \cdot d \\ \text{while } \left| x_1 - x_0 \right| > 1 \cdot mm \\ \quad \left| \begin{array}{l} x_0 \leftarrow \dfrac{x_0 + x_1}{2} \\ z \leftarrow d - e_c \\ F_c \leftarrow \dfrac{M_{s.max}}{z} \\ \sigma_{cc} \leftarrow \dfrac{F_c}{A_c} \\ F_s \leftarrow F_c + N_{tillh.1} \\ \sigma_s \leftarrow \dfrac{F_s}{A'_p} \\ x_1 \leftarrow \dfrac{\alpha \sigma_{cc}}{\sigma_s + \alpha \sigma_{cc}} \cdot d \end{array} \right. \end{array} \right.$$

Minimal moment:

$$X_{min} = \left| \begin{array}{l} x_0 \leftarrow 0.5H \\ z \leftarrow d - e_c \\ F_c \leftarrow \dfrac{M_{s.min}}{z} \\ \sigma_{cc} \leftarrow \dfrac{F_c}{A_c} \\ F_s \leftarrow F_c + N_{tillh.2} \\ \sigma_s \leftarrow \dfrac{F_s}{A'_p} \\ x_2 \leftarrow \dfrac{\alpha\sigma_{cc}}{\sigma_s + \alpha\sigma_{cc}} \cdot d \\ \text{while } |x_1 - x_0| > 1 \cdot mm \\ \quad \left| \begin{array}{l} x_0 \leftarrow \dfrac{x_0 + x_1}{2} \\ z \leftarrow d - e_c \\ F_c \leftarrow \dfrac{M_{s.min}}{z} \\ \sigma_{cc} \leftarrow \dfrac{F_c}{A_c} \\ F_s \leftarrow F_{c2} + N_{tillh.2} \\ \sigma_s \leftarrow \dfrac{F_s}{A'_p} \\ x_1 \leftarrow \dfrac{\alpha\sigma_{cc}}{\sigma_s + \alpha\sigma_{cc}} \cdot d \end{array} \right. \end{array} \right.$$

Summary location neutral axis:

Section	X_{max}	X_{min}	
1	711	61	←"field"
2	711	61	←"support"
-	mm	mm	

Function - calculate concrete stress (stadium II)

Maximum moment

$$\sigma_{cc.max} = \left| \begin{array}{l} z \leftarrow d - e_c \\ F_c \leftarrow \dfrac{M_{s.max}}{z} \\ \sigma_{cc} \leftarrow \dfrac{F_c}{A_c} \end{array} \right.$$

Minimum moment:

$$\sigma_{cc.min} = \left| \begin{array}{l} z \leftarrow d - e_c \\ F_c \leftarrow \dfrac{M_{s.min}}{z} \\ \sigma_{cc} \leftarrow \dfrac{F_c}{A_c} \end{array} \right.$$

Summary:

Section	$\sigma_{cc.max}$	$\sigma_{cc.min}$	
1	7	498	←"field"
2	7	498	←"support"
-	MPa	MPa	

Function - calculating design concrete stress

Maximum moment:

$$\sigma_{ök.1} = \begin{cases} |\sigma_{cö.1}| & \text{if } \sigma_{cu.1} < \sigma_{ct.till} \wedge \sigma_{cö.1} < 0MPa \\ \sigma_{cc.max} & \text{if } \sigma_{cu.1} > \sigma_{ct.till} \wedge \sigma_{cö.1} < 0MPa \\ 0MPa & \text{otherwise} \end{cases}$$

$$\sigma_{uk.1} = \begin{cases} |\sigma_{cu.1}| & \text{if } \sigma_{cö.1} < \sigma_{ct.till} \wedge \sigma_{cu.1} < 0MPa \\ \sigma_{cc.max} & \text{if } \sigma_{cö.1} > \sigma_{ct.till} \wedge \sigma_{cu.1} < 0MPa \\ 0MPa & \text{otherwise} \end{cases}$$

Section	$\sigma_{ök.1}$	$\sigma_{uk.1}$	
1	7	0	← "field"
2	7	0	← "support"
-	MPa	MPa	

Minimal moment:

$$\sigma_{ök.2} = \begin{cases} |\sigma_{cö.2}| & \text{if } \sigma_{cu.2} < \sigma_{ct.till} \wedge \sigma_{cö.2} < 0MPa \\ \sigma_{cc.min} & \text{if } \sigma_{cu.2} > \sigma_{ct.till} \wedge \sigma_{cö.2} < 0MPa \\ 0MPa & \text{otherwise} \end{cases}$$

$$\sigma_{uk.2} = \begin{cases} |\sigma_{cu.2}| & \text{if } \sigma_{cö.2} < \sigma_{ct.till} \wedge \sigma_{cu.2} < 0MPa \\ \sigma_{cc.min} & \text{if } \sigma_{cö.2} > \sigma_{ct.till} \wedge \sigma_{cu.2} < 0MPa \\ 0MPa & \text{otherwise} \end{cases}$$

Section	$\sigma_{ök.2}$	$\sigma_{uk.2}$	
1	1	4	← "field"
2	1	4	← "support"
-	MPa	MPa	

Function - calculate stress prestress (stadium II):

Maximum moment:

$$\sigma_{s.max} = \left| \begin{array}{l} z \leftarrow d - e_c \\ F_c \leftarrow \dfrac{M_{max}}{z} \\ F_s \leftarrow F_c + N_{tillh.1} \\ \sigma_s \leftarrow \dfrac{F_s}{A'_p} \end{array} \right.$$

Minimum moment:

$$\sigma_{s.min} = \left| \begin{array}{l} z \leftarrow d - e_c \\ F_c \leftarrow \dfrac{M_{min}}{z} \\ F_s \leftarrow F_c + N_{tillh.2} \\ \sigma_s \leftarrow \dfrac{F_s}{A'_p} \end{array} \right.$$

Summary:

Snitt	$\sigma_{s.max}$	$\sigma_{s.min}$	
1	34	5745	←"fält"
2	34	5745	←"stöd"
-	MPa	MPa	

Function - calculate steel stress in pretension cable

Maximum moment:

$$\sigma_{s.1} = \begin{cases} \sigma_{s.max} & \text{if } \sigma_{cp.1} \geq 0\text{MPa} \\ \sigma_{cp.1} \cdot \alpha_e & \text{if } \sigma_{cp.1} < 0\text{MPa} \end{cases}$$

Minimal moment:

$$\sigma_{s.2} = \begin{cases} \sigma_{s.min} & \text{if } \sigma_{cp.2} \geq 0\text{MPa} \\ \sigma_{cp.2} \cdot \alpha_e & \text{if } \sigma_{cp.2} < 0\text{MPa} \end{cases}$$

Summary:

Section	$\sigma_{s.1}$	$\sigma_{s.2}$	
1	34	-22	←"field"
2	34	-22	←"support"
-	MPa	MPa	

RESULTS

Verification of variation of stress in pretensioned tendons

Snitt	σ_{s1}	σ_{s2}	$\Delta\sigma_s$		$\Delta\sigma_{Rsd}$
1	34	-22	56	<	1391
2	34	-22	56	<	1391
-	MPa	MPa	MPa		MPa

Verication proves satisfactory resistance !

Verification of concrete stress
(See SS-EN 1992-1-1 section 6.8.7)

Results at "top" of beam:

$$E_{cd.max.equ.ÖK} = \frac{\max(\sigma_{ök.1}, \sigma_{ök.2})}{f_{cd.fat}}$$

$$E_{cd.min.equ.ÖK} = \frac{\min(\sigma_{ök.1}, \sigma_{ök.2})}{f_{cd.fat}}$$

$$R_{equ.ÖK} = \frac{E_{cd.min.equ.ÖK}}{E_{cd.max.equ.ÖK}}$$

$$\eta_{1.ÖK} = E_{cd.max.equ.ÖK} + 0.43 \cdot \sqrt{1 - R_{equ.ÖK}}$$

$$\eta_{2.ÖK} = E_{cd.max.equ.ÖK} + 0.50 - E_{cd.min.equ.ÖK}$$

Section	$\sigma_{ök.1}$	$\sigma_{ok.2}$	$f_{cd.fat}$	$E_{cd.max.equ}$	$E_{cd.min.equ}$	R_{equ}	η_1	η_2	
1	6,9	0,9	20,1	0,35	0,04	0,13	0,75	0,80	←"field"
2	6,9	0,9	23,3	0,30	0,04	0,13	0,70	0,76	←"support'
-	MPa	MPa	MPa	-	-	-	-	-	

Since $\eta_1 < 1.00$ and $\eta_2 < 1.00$ for all sections, resistance is satisfactory !

Results at "bottom" of beam:

$$E_{cd.max.equ.UK} = \frac{\max(\sigma_{uk.1}, \sigma_{uk.2})}{f_{cd.fat}}$$

$$E_{cd.min.equ.UK} = \frac{\min(\sigma_{uk.1}, \sigma_{uk.2})}{f_{cd.fat}}$$

$$R_{equ.UK} = \frac{E_{cd.min.equ.UK}}{E_{cd.max.equ.UK}}$$

$$\eta_{1.UK} = E_{cd.max.equ.UK} + 0.43 \cdot \sqrt{1 - R_{equ.UK}}$$

$$\eta_{2.UK} = E_{cd.max.equ.UK} + 0.50 - E_{cd.min.equ.UK}$$

Section	$\sigma_{uk.1}$	$\sigma_{uk.2}$	$f_{cd.fat}$	$E_{cd.max.equ}$	$E_{cd.min.equ}$	R_{equ}	η_1	η_2	
1	0,0	3,9	20,1	0,19	0,00	0,00	0,62	0,69	←"field"
2	0,0	3,9	23,3	0,17	0,00	0,00	0,60	0,67	←"support'
-	MPa	MPa	MPa	-	-	-	-	-	

Since $\eta_1 < 1.00$ and $\eta_2 < 1.00$ for all sections, resistance is satisfactory !