Object: Beam L1 (verification of principal stress)

THEORY

Work procedure for determination of principal tensile stress in concrete adjacent to prestressing tendon for Lk SLS:

For determination of the principal tensile stress, section constants are required. Section constants needed to determine bending tensile stresses and shear stresses associated with torsion (T) are entered directly into the program. Section constants required to determine shear stresses associated with transverse shear (Q) are calculated by program.

For the check, section constants denoted Type 1 are used for stresses associated with prestressing. For other loads, section constants Type 2 are used.

Section constants Type 1: ungrouted section
Section constants Type 2: grouted section
The principal tensile stress (σ_I) comprises terms for bending tensile stress (σ), shear stresses associated with transverse shear (τ_Q) and torsion (τ_T) according to the formulas below.

$$\sigma(q) = \frac{M}{I_c} \cdot (q - y_{tp}) + \frac{N}{A_c}$$

$$\tau_Q(q) = \frac{S(q)}{I_c \cdot t(t)} \cdot Q \qquad \tau_T = \frac{1}{W_v} \cdot T$$

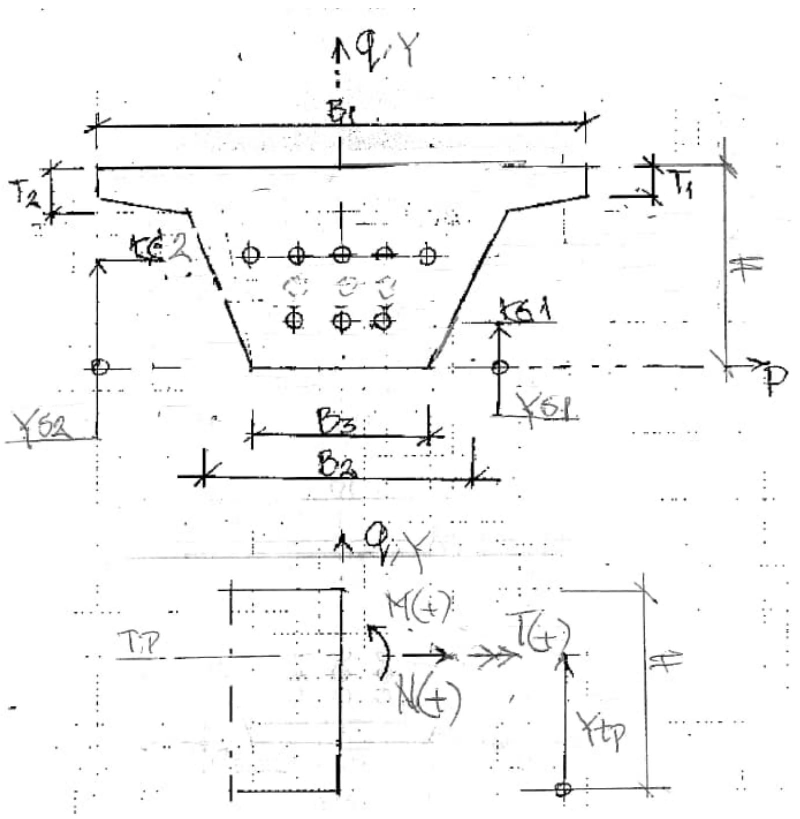
$$\sigma_I = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + (\tau_Q + \tau_T)^2}$$

$\sigma_{I,\,ÖK}$: principal tensile stress at top tendon level + 0.100 m

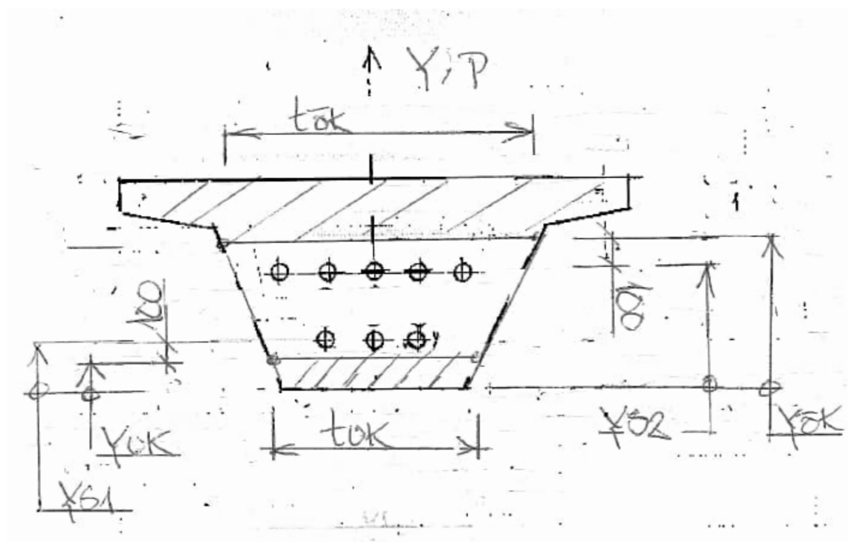
$\sigma_{I,\,UK}$: principal tensile stress at bottom tendon level − 0.100 m

PRINCIPAL SKETCH

Definition cross section and forces



Definition of variables calculation



INPUT

Number of sections

$N := 7 \cdot st$

Cross section properties

Ungrouted cross section (type 1):

Section	A_{c1}	I_{c1}	y_{tp1}	$W_{v1.ök}$	$W_{v1.uk}$
1	4,428	0,813	0,819	1,567	2,611
2	4,428	0,813	0,819	1,567	2,611
3	4,428	0,813	0,819	1,567	2,611
4	4,428	0,813	0,819	2,447	1,501
5	4,428	0,813	0,819	1,500	2,432
6	4,428	0,813	0,819	1,500	2,432
7	4,428	0,813	0,819	1,500	2,432
-	m^2	m^4	m	m^3	m^3

$W_{v1.min}$
1,119
1,119
1,119
1,119
1,119
1,119
1,119
m^3

Grouted cross section (type 1):

Section	A_{c2}	I_{c2}	y_{tp2}	$W_{v2.ök}$	$W_{v2.uk}$
1	4,428	0,813	0,819	1,567	2,611
2	4,428	0,813	0,819	1,567	2,611
3	4,428	0,813	0,819	1,567	2,611
4	4,428	0,813	0,819	2,447	1,501
5	4,428	0,813	0,819	1,500	2,432
6	4,428	0,813	0,819	1,500	2,432
7	4,428	0,813	0,819	1,500	2,432
-	m^2	m^4	m	m^3	m^3

$W_{v2.min}$
1,119
1,119
1,119
1,119
1,119
1,119
1,119
m^3

Geometry (basis for calculation of τ_Q)
Flange effects are handled the same as for torsion — not considered.

Section	B_1	B_2	B_3	H	T_1	T_2
1	2500	2500	2500	1400	300	300
2	2500	2500	2500	1400	300	300
3	2500	2500	2500	1400	300	300
4	2500	2500	2500	1400	300	300
5	2500	2500	2500	1400	300	300
6	2500	2500	2500	1400	300	300
7	2500	2500	2500	1400	300	300
-	mm	mm	mm	mm	mm	mm

Cable layout (max. 2 cable layers may be considered)

The position of the upper cable group is denoted: y_{s2}
The position of the lower cable group is denoted: y_{s1}

Section	A_s	n_1	y_{s1}	n_2	y_{s2}
1	2250	6	1,100	6	1,100
2	2250	6	1,100	6	1,100
3	2250	6	1,100	6	1,100
4	2250	6	0,280	6	0,280
5	2250	6	1,122	6	1,122
6	2250	6	1,122	6	1,122
7	2250	6	1,122	6	1,122
-	mm^2	st	m	st	m

Forces

Pretension load at time t_0 & prestress losses at time t_1 and t_2 :

			Time - t_0			
Section	η_{t1}	η_{t2}	$M_{FÖRSP}$	$N_{FÖRSP}$	$Q_{FÖRSP}$	$T_{FÖRSP}$
1	6%	16,0%	17210	-30329	-664	-12
2	6%	16,0%	17210	-30329	-664	-12
3	6%	16,0%	17210	-30329	-664	-12
4	6%	16,0%	-12955	-31780	155	-19
5	6%	16,0%	17698	-30122	-1705	43
6	6%	16,0%	17698	-30122	-1705	43
7	6%	16,0%	17698	-30122	0	43
-	-	-	kNm	kN	kN	kNm

Load combination SLS (excl. pretension) :

Section	M_{SLS}	N_{SLS}	Q_{SLS}	T_{SLS}	Remark
1	-11322	-3137	2307	1117	Min M
2	-8308	-2169	2076	2083	Max T
3	-9840	-2954	2554	1047	Max Q
4	9296	-2266	-115	520	Max M
5	-13945	-1716	-2236	-2057	Min M
6	-11927	-1540	-2103	-2497	Min T
7	-13267	-1991	-2476	-1593	Min Q
-	kNm	kN	kN	kNm	-

CALCULATION

Level top tendon + 100 mm (denoted "ök")

$$y_{ök} := y_{s2} + 0.1 \cdot m$$

Level bottom tendon - 100 mm (denoted "uk")

$$y_{uk} := y_{s1} - 0.1 \cdot m$$

Function - p:coordinate as function of q:koordinat

$$p(q) := \begin{cases} 0.5 \cdot \left[B_3 + (B_2 - B_3) \cdot \dfrac{q}{H - T_2}\right] & \text{if } q \le (H - T_2) \\ 0.5 \cdot \left[B_2 + (B_1 - B_2) \cdot \dfrac{q - H + T_2}{T_2 - T_1}\right] & \text{if } (H - T_2) < q \le (H - T_1) \\ 0.5 \cdot B_1 & \text{if } (H - T_1) < q \le H \end{cases}$$

Function - static moment at "bottom" of grouted section

$$S_{uk} := \begin{cases} 2 \displaystyle\int_{0m}^{y_{uk}} p(q) \cdot (y_{tp2} - q) \, dq & \text{if } y_{uk} \le y_{tp2} \\ 2 \displaystyle\int_{y_{uk}}^{H} p(q) \cdot (H + y_{tp2} - q) \, dq & \text{otherwise} \end{cases}$$

Function - static moment at "top" of grouted section

$$S_{ök} := \begin{cases} 2 \displaystyle\int_{y_{ök}}^{H} p(q) \cdot (H + y_{tp2} - q) \, dq & \text{if } y_{ök} \ge y_{tp2} \\ 2 \displaystyle\int_{0m}^{y_{ök}} p(q) \cdot (y_{tp2} - q) \, dq & \text{otherwise} \end{cases}$$

Function - thickness att "bottom"

$$t_{uk} := 2 \cdot p(y_{uk})$$

Function - thickness at "top"

$$t_{ök} := 2 \cdot p(y_{ök})$$

Function - constant shear flow at "bottom"

$$k_{uk} := \frac{S_{uk}}{I_{c2} \cdot t_{uk}}$$

Function - constant shear flow at "top"

$$k_{ök} := \frac{S_{ök}}{I_{c2} \cdot t_{ök}}$$

Principal tensile stress

Time t_0 :

Bending-tensile stress:

$$\eta_{t0} = 0 \cdot \%$$

$$\sigma_{\ddot{o}k.0} = -\frac{M_{F\ddot{O}RSP_i} \cdot (1 - \eta_{t0})}{I_{c1}} \cdot (y_{\ddot{o}k} - y_{tp1}) - \frac{M_{SLS}}{I_{c2}} \cdot (y_{\ddot{o}k} - y_{tp2}) \ldots + \frac{N_{F\ddot{O}RSP} \cdot (1 - \eta_{t0})}{A_{c1}} + \frac{N_{SLS}}{A_{c2}}$$

$$\sigma_{uk.0} = \frac{M_{F\ddot{O}RSP} \cdot (1 - \eta_{t0})}{I_{c1}} \cdot (y_{tp1} - y_{uk}) + \frac{M_{SLS}}{I_{c2}} \cdot (y_{tp2} - y_{uk}) \ldots + \frac{N_{F\ddot{O}RSP} \cdot (1 - \eta_{t0})}{A_{c1}} + \frac{N_{SLS}}{A_{c2}}$$

Shear force stress:

$$\tau_{Q.\ddot{o}k.0} = k_{\ddot{o}k} \cdot \left[Q_{SLS} + Q_{F\ddot{O}RSP} \cdot (1 - \eta_{t0}) \right]$$

$$\tau_{Q.uk.0} = k_{uk} \cdot \left[Q_{SLS} + Q_{F\ddot{O}RSP} \cdot (1 - \eta_{t0}) \right]$$

Torsional stress:

$$\tau_{T.\ddot{o}k.0} = \frac{T_{F\ddot{O}RSP} \cdot (1 - \eta_{t0})}{W_{v1.\ddot{o}k}} + \frac{T_{SLS}}{W_{v2.\ddot{o}k}}$$

$$\tau_{T.uk.0} = \frac{T_{F\ddot{O}RSP} \cdot (1 - \eta_{t0})}{W_{v1.uk}} + \frac{T_{SLS}}{W_{v2.uk}}$$

Principle tensile stress:

$$\sigma_{t0.uk} = \frac{\sigma_{uk.0}}{2} + \sqrt{\frac{\sigma_{uk.0}^{\;2}}{4} + \left(|\tau_{Q.uk.0}| + |\tau_{T.uk.0}| \right)^2}$$

$$\sigma_{t0.\ddot{o}k} = \frac{\sigma_{\ddot{o}k.0}}{2} + \sqrt{\frac{\sigma_{\ddot{o}k.0}^{\;2}}{4} + \left(|\tau_{Q.\ddot{o}k.0}| + |\tau_{T.\ddot{o}k.0}| \right)^2}$$

Time t_1 :

Bending-tensile stress:

$$\sigma_{ök.1} = -\frac{M_{FÖRSP}\cdot(1-\eta_{t1})}{I_{c1}}\cdot(y_{ök}-y_{tp1}) - \frac{M_{SLS}}{I_{c2}}\cdot(y_{ök}-y_{tp2}) \ldots + \frac{N_{FÖRSP}\cdot(1-\eta_{t1})}{A_{c1}} + \frac{N_{SLS}}{A_{c2}}$$

$$\sigma_{uk.1} = \frac{M_{FÖRSP}\cdot(1-\eta_{t1})}{I_{c1}}\cdot(y_{tp1}-y_{uk}) + \frac{M_{SLS}}{I_{c2}}\cdot(y_{tp2}-y_{uk}) \ldots + \frac{N_{FÖRSP}\cdot(1-\eta_{t1})}{A_{c1}} + \frac{N_{SLS}}{A_{c2}}$$

Shear force stress:

$$\tau_{Q.ök.1} = k_{ök}\cdot\left[Q_{SLS} + Q_{FÖRSP}\cdot(1-\eta_{t1})\right]$$

$$\tau_{Q.uk.1} = k_{uk}\cdot\left[Q_{SLS} + Q_{FÖRSP}\cdot(1-\eta_{t1})\right]$$

Torsional stress:

$$\tau_{T.ök.1} = \frac{T_{FÖRSP}\cdot(1-\eta_{t1})}{W_{v1.ök}} + \frac{T_{SLS}}{W_{v2.ök}}$$

$$\tau_{T.uk.1} = \frac{T_{FÖRSP}\cdot(1-\eta_{t1})}{W_{v1.uk}} + \frac{T_{SLS}}{W_{v2.uk}}$$

Principle tensile stress:

$$\sigma_{t1.uk} = \frac{\sigma_{uk.1}}{2} + \sqrt{\frac{\sigma_{uk.1}^{\;2}}{4} + \left(\left|\tau_{Q.uk.1}\right| + \left|\tau_{T.uk.1}\right|\right)^2}$$

$$\sigma_{t1.ök} = \frac{\sigma_{ök.1}}{2} + \sqrt{\frac{\sigma_{ök.1}^{\;2}}{4} + \left(\left|\tau_{Q.ök.1}\right| + \left|\tau_{T.ök.1}\right|\right)^2}$$

Time t_2 :

Bending-tensile stress:

$$\sigma_{ök.2} = -\frac{M_{FÖRSP}\cdot(1-\eta_{t2})}{I_{c1}}\cdot(y_{ök}-y_{tp1}) - \frac{M_{SLS}}{I_{c2}}\cdot(y_{ök}-y_{tp2}) \ldots + \frac{N_{FÖRSP}\cdot(1-\eta_{t2})}{A_{c1}} + \frac{N_{SLS}}{A_{c2}}$$

$$\sigma_{uk.2} = \frac{M_{FÖRSP}\cdot(1-\eta_{t2})}{I_{c1}}\cdot(y_{tp1}-y_{uk}) + \frac{M_{SLS}}{I_{c2}}\cdot(y_{tp2}-y_{uk}) \ldots + \frac{N_{FÖRSP}\cdot(1-\eta_{t2})}{A_{c1}} + \frac{N_{SLS}}{A_{c2}}$$

Shear force stress:

$$\tau_{Q.ök.2} = k_{ök}\cdot\left[Q_{SLS} + Q_{FÖRSP}\cdot(1-\eta_{t2})\right]$$

$$\tau_{Q.uk.2} = k_{uk}\cdot\left[Q_{SLS} + Q_{FÖRSP}\cdot(1-\eta_{t2})\right]$$

Torsional stress:

$$\tau_{T.ök.2} = \frac{T_{FÖRSP}\cdot(1-\eta_{t2})}{W_{v1.ök}} + \frac{T_{SLS}}{W_{v2.ök}}$$

$$\tau_{T.uk.2} = \frac{T_{FÖRSP}\cdot(1-\eta_{t2})}{W_{v1.uk}} + \frac{T_{SLS}}{W_{v2.uk}}$$

Principle tensile stress:

$$\sigma_{t2.uk} = \frac{\sigma_{uk.2}}{2} + \sqrt{\frac{\sigma_{uk.2}^{\,2}}{4} + \left(\left|\tau_{Q.uk.2}\right| + \left|\tau_{T.uk.2}\right|\right)^2}$$

$$\sigma_{t2.ök} = \frac{\sigma_{ök.2}}{2} + \sqrt{\frac{\sigma_{ök.2}^{\,2}}{4} + \left(\left|\tau_{Q.ök.2}\right| + \left|\tau_{T.ök.2}\right|\right)^2}$$

RESULTS

Calculated cross-sectional properties

Section	y_{uk}	t_{uk}	S_{uk}	k_{uk}	$y_{ök}$	$t_{ök}$	$S_{ök}$	$k_{ök}$
1	1000	2500	1,019	0,501	1200	2500	0,460	0,226
2	1000	2500	1,019	0,501	1200	2500	0,460	0,226
3	1000	2500	1,019	0,501	1200	2500	0,460	0,226
4	180	2500	0,328	0,161	380	2500	0,598	0,294
5	1022	2500	0,953	0,469	1222	2500	0,404	0,199
6	1022	2500	0,953	0,469	1222	2500	0,404	0,199
7	1022	2500	0,953	0,469	1222	2500	0,404	0,199
-	mm	mm	m^3	$1/m^2$	mm	mm	m^3	$1/m^2$

Bending-tensile stresses

	t_0		t_1		t_2	
Section	σ_{uk}	$\sigma_{ök}$	σ_{uk}	$\sigma_{ök}$	σ_{uk}	$\sigma_{ök}$
1	-8,87	-10,32	-8,23	-9,42	-7,16	-7,93
2	-9,32	-11,51	-8,68	-10,62	-7,61	-9,12
3	-9,16	-10,97	-8,52	-10,08	-7,45	-8,58
4	-10,56	-9,66	-9,52	-8,81	-7,79	-7,40
5	-8,13	-9,05	-7,45	-8,12	-6,33	-6,56
6	-8,59	-10,01	-7,92	-9,08	-6,80	-7,52
7	-8,36	-9,45	-7,69	-8,51	-6,56	-6,96
-	MPa	MPa	MPa	MPa	MPa	MPa

Principle tensile stresses

	t_0		t_1		t_2	
Section	$\sigma_{I,uk}$	$\sigma_{I,ök}$	$\sigma_{I,uk}$	$\sigma_{I,ök}$	$\sigma_{I,uk}$	$\sigma_{I,ök}$
1	0,17	0,11	0,19	0,12	0,23	0,15
2	0,24	0,23	0,26	0,25	0,31	0,29
3	0,19	0,11	0,21	0,12	0,25	0,14
4	0,01	0,00	0,01	0,01	0,01	0,01
5	0,80	0,47	0,83	0,51	0,90	0,60
6	0,83	0,54	0,86	0,58	0,92	0,67
7	0,37	0,24	0,40	0,27	0,46	0,32
-	MPa	MPa	MPa	MPa	MPa	MPa